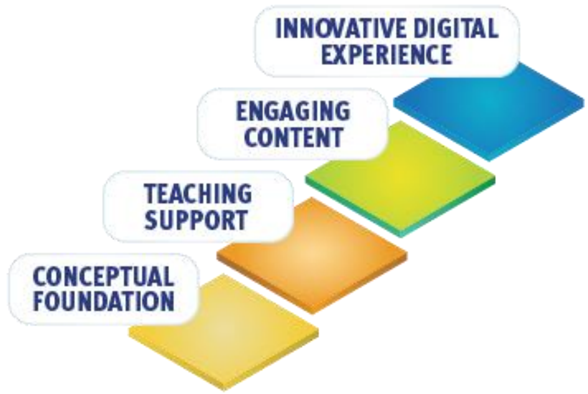
Math & YOU Research Foundation

Written by renowned author, Dr. Ron Larson, and his expert authorship team, Math & YOU is a comprehensive mathematics program grounded in research and developed around four central pillars, namely: Conceptual Foundation, Engaging Content, Teaching Support, and Innovative Platform. Together, these four pillars focus the program on YOU, the student and the teacher, to provide an integrated experience that empowers teachers to enhance student learning.

This document provides a description of the research informing the development of the Math & YOU program. Organized around the four pillars of the program, this paper provides a description of research informing each pillar followed by a description of how the research is evident in the Math & YOU program across the K-5, 6-8, and Algebra 1, Geometry, Algebra 2 (AGA) materials.



Math & YOU Pillars:

- **Conceptual Foundation** recognizes mathematical rigor as a balance of procedural fluency, conceptual understanding, and application. Building from a foundation of conceptual understanding strengthens procedural fluency by expanding and connecting students' repertoire of meaningful strategies. Research-based accounts of developmental progressions of thinking and instructional sequences to support advancement of thinking are leveraged in building Math & YOU content to ensure students develop a coherent understanding that connects and builds across lessons, chapters, and grade levels.
- **Engaging Content** attends to students' cognitive and emotional engagement with the mathematics presented. Cognitive engagement can be achieved through lessons that get students actively involved and incorporate the standards for mathematical practice. Emotional engagement supports students' views of mathematics as useful and themselves as doers of mathematics. As students develop these mindsets, they are more likely to develop grit and engage in productive struggle, keys to mathematical success.
- **Teaching Support** means providing resources that empower teachers and ultimately enhance student learning. While this begins with a lesson design that encourages use of research-based effective teaching practices, it is enhanced through features that support in-the-moment decision making grounded in knowledge of the content and of pedagogy to support students' understanding of the content.
- **Innovative Platform** enhances the other three pillars by providing more customization of content for students' specific learning needs, more content interactivity and individualized feedback to strengthen engagement, and timely and informative data to support teachers in instructional decision-making.

Pillar I: Conceptual Foundation

Given Big Ideas Learning's singular focus on mathematics content, supporting students' development of a strong conceptual foundation of mathematics naturally drove the development of the Math & YOU program. Beginning each lesson by developing students' understanding of key concepts and building connections to prior understanding, Math & YOU equips students with a strong foundation for developing procedural fluency and application of problems in new situations. Furthermore, building understanding of mathematics as coherent and important supports students' development of productive mathematical dispositions.

> "Students with conceptual understanding know more than isolated facts and methods. They understand why a mathematical idea is important and the kinds of contexts in which it is useful. They have organized their knowledge into a coherent whole, which enables them to learn new ideas by connecting those ideas to what they already know."
>
> National Research Council, 2001, p. 118

What Research Informs the Pillar?

What does it mean to be mathematically proficient? According to the National Research Council (NRC, 2001), mathematical proficiency involves five interwoven strands of mathematical competency, namely procedural fluency (i.e., skill in carrying out procedures), conceptual understanding (i.e., comprehension of concepts and relations), strategic competence (i.e., ability to formulate, represent and solve problems), adaptive reasoning (i.e., capacity for logical thought, reflection, explanation and justification), and productive disposition (i.e., seeing mathematics as useful and oneself as mathematically capable). Importantly, the NRC emphasizes that mathematical proficiency cannot be achieved by focusing on just one or two of these strands, rather mathematics programs must attend to all five strands for students to become increasingly proficient in mathematics (NRC, 2001). More than twenty years after the original definition of the five strands of mathematical proficiency, debates still ensue over whether procedural fluency or conceptual understanding is more important. Yet the research is clear and supports the stance taken by the NRC two decades ago. Conceptual understanding and procedural fluency (along with strategic competence, adaptive reasoning, and productive disposition) are both equally important and necessary components of students' mathematical understanding (National Council of Teachers of Mathematics, 2023).

Seeking to more thoroughly define a "deep understanding" of mathematics, the Common Core State Standards for Mathematics define *rigorous* mathematical understanding as including three aspects with equal intensity, namely procedural fluency, conceptual understanding, and application (National Governor's Association (NGA) Center for Best Practices and Council of Chief State School Officers (CCSSO), 2010). Informed by a breadth of research showing the foundational role of conceptual understanding for building students' procedural fluency with mathematics (Hiebert & Grouws, 2007; Rittle-Johnson, Schneider & Star, 2015), the National Council of Teachers of Mathematics (NCTM, 2014, 2023) identified "building procedural fluency from conceptual understanding" as one of eight effective teaching practices. NCTM (2023) elaborated on this teaching practice, explaining the dangers of teaching procedures prior to a conceptual foundation (e.g., more prone to making errors, unable to identify nonsensical answers) and illustrating how a conceptual foundation can lead to opportunities to develop additional strategies rather than rote memorization of step-by-step algorithms.

Building experiences that support students' conceptual development across the curriculum requires coherence (NGA & CCSSO, 2010; NCTM, 2014). For particular mathematical content, learning trajectories can provide a description of learning goals, developmental progressions of thinking and learning, and sequences of instructional tasks to encourage students to progress to advanced levels of understanding (Clements & Sarama, 2011, 2009; Simon 1995; Simon & Tzur, 2004). Such an approach aligns with efforts to engage students in productive struggle (NCTM, 2014; Warshauer, 2015) by providing experiences and scaffolding to help students advance along the hypothetical learning trajectory. These sequences of experiences, or learning progressions, provide a description of the "successively more sophisticated ways of thinking that can follow one another as children learn about and investigate a topic" (NRC, 2007, p. 214). Learning progressions should inform curriculum development since instructional pathways should be designed to help students advance through the progression and should inform pedagogy as teachers use learning progressions to evaluate student progress and make instructional decisions based on elicited thinking (Battista, 2011; Cutting & Lowrie, 2021; Martínez et al., 2022).

With the lessons centered around research-based, developmental progressions of content, teachers and students are empowered to monitor progress toward the learning goals of the lesson. But this monitoring requires the research-supported practice of clearly focusing the lesson and communicating the lesson goals so that it can be shared with students in an easy-to-understand format (Hattie, 2012, 2023; Hattie, Fisher, & Frey, 2017; Marzano, 2007). Sharing success criteria, or student-friendly language that supports students' self-monitoring of their progress toward the lesson's learning goals, coupled with frequent self-assessment opportunities, are proven strategies that help to accelerate student learning (Hattie, 2012, 2023; Hattie, Fisher & Frey, 2017).

How is the Pillar Visible within Math & YOU?

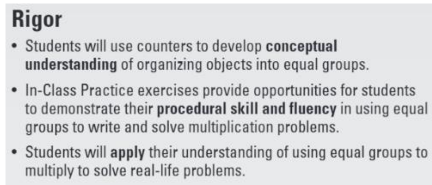
The foundational pillar of building a strong **Conceptual Foundation** can be seen in the lesson structure and learning progressions throughout the Math & YOU program.

Lesson Structure

Each lesson in Math & YOU was thoughtfully developed with a focus on building mathematical rigor (NGA & CCSSO, 2010). The K-5 and 6-8 Teaching Editions explicitly identify the emphasis on conceptual understanding, procedural skills and fluency, and application in the Section Overview at the start of each lesson, as illustrated in Figure 1. This alerts the teacher to the three important elements of rigor and indicates what aspects of the current lesson will support students' development of each component.

Figure 1

Description of Mathematical Rigor Within a Lesson (Math & YOU Grade 3 TE Vol. 1, p. 3A)



Rigor

- Students will use counters to develop **conceptual understanding** of organizing objects into equal groups.
- In-Class Practice exercises provide opportunities for students to demonstrate their **procedural skill and fluency** in using equal groups to write and solve multiplication problems.
- Students will **apply** their understanding of using equal groups to multiply to solve real-life problems.

But this description of rigorous learning outcomes at the outset of the lesson is just the start. Every K-5, 6-8, and AGA lesson is purposefully designed to support students' development of conceptual understanding, procedural skill and fluency, and application of the content. A description of the consistent lesson features across Math & YOU and their relationship to the aspects of rigor is provided in Table 1, with further discussion of how they support *rigor* in the following paragraphs. These lesson features are carefully sequenced to ensure that students are building procedural fluency from conceptual understanding, as emphasized by NCTM (2014, 2023) while providing a balance of the aspects of rigor.

Table 1

Math & YOU Lesson Components Designed to Balance Three Aspects of Mathematical Rigor

	K-5	**6-8**	**AGA**
Conceptual Understanding	*Investigate* *Key Concept* *In-Class Practice: Reasoning*	*Investigate* *Key Concept*	*Investigate* *Key Concept*
Procedural Skill and Fluency	*Key Concept* *In-Class Practice* *Practice*	*Key Concept* *In-Class Practice* *Practice*	*Key Concept* *In-Class Practice* *Practice*
Application	*Big Idea of the Chapter* *Connect to Real Life* within *In-Class Practice* *Performance Task* *Connecting Big Ideas*	*Big Idea of the Chapter* *Connecting to Real Life* within *In-Class Practice* *Performance Task* *Connecting Big Ideas*	*Big Idea of the Chapter* *Connecting to Real Life* within *In-Class Practice* *Interpreting Data* within *Practice* *Performance Task* *Connecting Big Ideas*

A student's learning journey begins at the chapter level with the Big Idea of the Chapter, an exploration of the ways the ideas of the chapter will be *applied* to solve problems relating to students' lives. Within each lesson, the Investigate activity starts each lesson with a *conceptual* investigation of the topic that encourages students to ask new questions and connect to prior understanding, both key aspects of conceptual understanding (NCTM, 2014; NRC, 2001). In K-5, this Investigate activity focuses on a look back and look forward approach while in 6-8 and AGA, the activity provides an opportunity for students to explore with carefully crafted assessing and advancing questions (Smith & Sherin, 2019) to support the teacher in accessing students' prior knowledge while opening a pathway to construction of extended and new ideas. This investigation builds seamlessly into the Key Concept of the lesson, where new concepts are formally presented and connected to *procedural* strategies as appropriate. Opportunities for In-Class Practice and Practice follow the Key Concept in K-5 and are interspersed throughout the Key Concept in 6-8 and AGA, allowing students to build accuracy, efficiency, and flexibility with strategies (NRC, 2001; NCTM, 2014, 2023). Throughout the Teacher Editions, Talk About It questions connect *procedural skill* and fluency to both *conceptual understanding* and *application* while working through In-Class Practice. Figure 2a provides an example from the 5th Grade Teacher Edition of Talk About It questions designed to encourage students to *apply* knowledge (to the context of volume of a cube) while gaining *procedural fluency* (with using arrays to multiply). Figure 2b provides an example from the 6th Grade Teacher Edition of questions designed to help students connect their *procedural* approaches for evaluating expressions back to their *conceptual* knowledge of mathematical properties which inform those approaches.

Figure 2

Talk About It Examples to Connect Procedural Fluency to Application and Conceptual Understanding

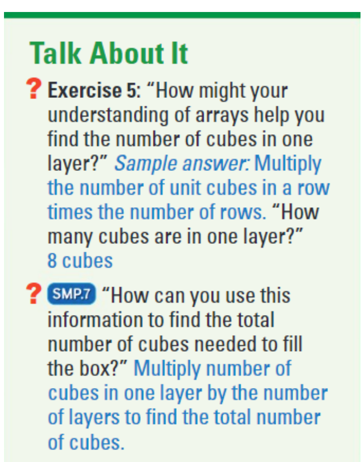


Fig 2a. Math & YOU Grade 5 TE, p. 561

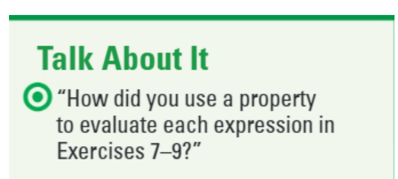


Fig 2b. Math & YOU Grade 6 TE, p. 18

Further opportunities to *apply* the new ideas are provided through the targeted Connections to Real Life tasks included within the In-Class Practice in all grade bands, and within the Interpreting Data tasks included in the AGA In-Class Practice. Connections between *procedural* strategies and *conceptual* understanding are further supported within the In Class Practice: Reasoning tasks provided in K-5. These problems are included in every lesson and are designed to engage students in thinking conceptually about the strategies they are using as they seek to extend and generalize the lesson content. At the chapter level, a Performance Task provides another opportunity to develop students' *application* of the knowledge learned across the chapter. At four points within each book, a Connecting Big Ideas task challenges students to *apply* their knowledge across the chapters to solve problems related to everyday life.

Learning Progressions supported by Focused Learning

Students' **Conceptual Foundation** of mathematics is also supported through the use of learning progressions during content development and through clear communication of these learning progressions throughout the Teacher Editions. Learning progressions are integral to the development of Math & YOU and inform each stage of content development. The authors gave specific attention to the details of the Standards progressions while developing the sequencing and instructional content for each course.

Learning progressions are also visible throughout the final product. At the outset of each chapter, the Standards for Content and Mathematical Practice describe the content standards addressed in each lesson via the Coherence through the Chapter feature (showing whether students are preparing to learn, learning, or extending learning, see Figure 3a). On a broader scale, learning progressions across grade levels are indicated in the Coherence through the Grades feature (see Figure 3b), which provides relevant learning that precedes or follows (whether in the current course or earlier grades) the learning happening within the current lesson. The Mathematics of the Chapter further orients teachers to the progressions of student learning by discussing how the content of the current chapter fits into students' overall mathematics understanding. Within each lesson, the Coherence description in the Section Overview provides information about how the lesson-specific learning target and success criteria align with previous and future learning. This feature provides teachers with a sequence of observable indicators of increasingly sophisticated understanding and skill for key concepts within each chapter. The Big Ideas of the Grade feature provides a grade-by-grade progressions chart for teachers to show how the big ideas of math across each grade connect and build as students move from grade-to-grade.

Learning is focused on clearly stated Learning Targets and Success Criteria, communicated both to the teacher and to the students at the beginning of each lesson (see Figure 3c). Furthermore, both the Teacher Edition and Student Edition provide support to monitor progress toward these learning outcomes. Teaching notes such as the one previously seen in Figure 2b provide a visual representation of a target to direct the teacher's attention to the important link between the idea provided in the note and the learning target for the lesson. Throughout the Student Edition, opportunities for self-assessment direct students' attention back to the learning target for the day's lesson so students can reflect on progress at the point-of-use. Supports for students and teachers throughout the lessons ensure that the learning target and success criteria are not simply stated at the beginning of the lesson but are revisited frequently during the learning process to ensure that students continue to make progress along the learning progressions that informed the lesson.

Providing a comprehensive view of the learning progression and how student understanding is likely to develop over time, as well as detailed lesson-level learning targets and success criteria, gives teachers a conceptual tool that can assist them in making targeted and specific lesson sequencing and support decisions while visibly mapping students' progress.

Figure 3

Coherence Features and Learning Targets and Success Criteria Showcase the Developmental Progression of Content Through the Chapter and Through the Grades (Grade 7 TE p. 0C-0D)

***COHERENCE* Through the Chapter**

Content Standard	1.1	1.2	1.3	1.4	1.5
Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line=diagram.	▲				
Describe situations in which opposite quantities combine to make 0.		●	●		
Understand $p + q$ as the number located a distance $\lvert q \rvert$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.		●	●		
Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.				●	●
Apply properties of operations as strategies to add and subtract rational numbers.		●	●		●
Solve real-world and mathematical problems involving the four operations with rational numbers.	▲	●	●	●	●

Key: ▲ preparing ● learning ★ extending

Fig 3a. Coherence Through the Chapter (Grade 7 TE, p. 0C)

COHERENCE Through the Grades

Prior Learning	Current Learning	Future Learning
Grade 4 • **Sections 2.8, 3.8, and 4.7:** Solve multi-step word problems using the four operations. **Grade 5** • **Chapter 8:** Add and subtract fractions and mixed numbers with unlike denominators. **Grade 6** • **Section 2.5:** Use standard algorithms to fluently add and subtract decimals. • **Sections 8.1 and 8.3:** Understand the concept of negative numbers and that they are used along with positive numbers to describe quantities. • **Sections 8.1 and 8.3:** Graph an integer or a rational number and its opposite on a number line. • **Sections 8.2 and 8.3:** Compare and order integers and rational numbers by graphing them on a number line. Interpret statements about order in real-life problems. • **Section 8.4:** Understand the concept of absolute value. Apply absolute value in real-life problems. • **Section 8.4:** Make comparisons that involve absolute values of numbers.	**Grade 7 Chapter 1** • **Section 1.1:** Graph rational numbers on a number line, find the absolute value of a rational number, and use a number line to compare rational numbers. • **Sections 1.2 and 1.3:** Explain how to model addition of integers or rational numbers on a number line. Explain why the sum of a number and its opposite is 0. • **Sections 1.2 and 1.3:** Find sums of integers or rational numbers by reasoning about absolute values. Interpret sums of rational numbers by describing real-life situations. • **Sections 1.2, 1.3, and 1.5:** Use properties of operations to add and subtract integers and rational numbers. • **Sections 1.2–1.5:** Solve mathematical and real-life problems involving adding and subtracting integers or rational numbers. • **Sections 1.2 and 1.3:** Describe situations in which opposite quantities combine to make 0. • **Sections 1.4 and 1.5:** Explain how subtracting integers is related to adding integers. Explain how to model subtraction of integers or rational numbers on a number line. • **Sections 1.4 and 1.5:** Find differences of integers or rational numbers by reasoning about absolute values. • **Section 1.5:** Find distances between two rational numbers on a number line.	**Grade 7** • **Sections 2.1 and 2.4:** Explain the rules for multiplying integers or rational numbers. Find products of integers or rational numbers. Interpret products of rational numbers by describing real-life situations. • **Sections 2.1 and 2.4:** Use properties of operations to multiply integers and rational numbers. • **Sections 2.1, 2.2, 2.4, and 2.5:** Solve mathematical and real-life problems involving multiplying and dividing integers or rational numbers. • **Sections 2.2, 2.3, and 2.5:** Explain the rules for dividing integers or rational numbers. Find quotients of integers or rational numbers. Interpret quotients of rational numbers by describing real-life situations. • **Sections 4.1–4.3:** Write and solve one- and two-step equations. • **Sections 4.4–4.7:** Write and solve one- and two-step inequalities. Graph the solution set of an inequality and interpret it within a real-life context. **Grade 8** • **Section 9.4:** Explain the meaning of rational numbers. Write fractions and mixed numbers as decimals. Write repeating decimals as fractions or mixed numbers. • **Section 9.5:** Understand the concept of irrational numbers. Know that $\sqrt{2}$ is irrational. • **Section 9.5:** Approximate irrational numbers and locate the approximations on a number line.

Fig 3b. Coherence Through the Grade (Grade 7 TE, p. 0D)

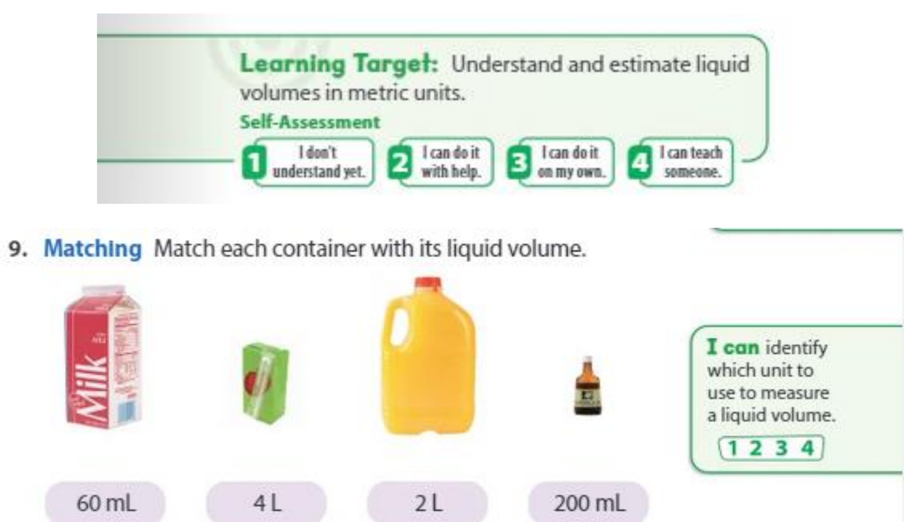


Fig 3c. Student-Facing Learning Target and Success Criteria for Self-Monitoring (Grade 3 SE, p. 457, 459)

Pillar II: Engaging Content

While building a curriculum to support procedural fluency from a rich conceptual foundation is imperative for developing students' robust understanding of mathematics, it is certainly not sufficient. Research indicates that students are becoming disengaged with mathematics at rates never seen before (Cooper, 2014; Lawson and Lawson, 2013). Without student engagement, even curricula with significant learning affordances will not have the intended outcomes (Henningsen & Stein, 1997). Thus, curricula must attend to delivering content in a student-centered approach that sets high expectations for all students and which subscribes to the belief that all students can learn mathematics.

> "Promoting student engagement, framing mathematics within the growth mindset, acknowledging student contributions, and attending to culture and language play substantial roles in equalizing mathematics gains between poor and non-poor students."
>
> NCTM, 2014, p. 65

What Research Informs the Pillar?

Student engagement is a broad idea that encompasses cognitive, behavioral, and emotional components (Fredricks, Blumenfeld, & Paris, 2004; Reeve et al., 2004), despite often being overgeneralized to behaviors such as staying on task or following directions as expected during instruction (Fredricks et al., 2004). In addition to these behavioral aspects, cognitive engagement is related to the amount of effort put forward in thinking about content and the response to challenges or setbacks during learning, while emotional engagement is related to a student's interest and positive feelings related to learning the content (Fredricks et al., 2004).

In response to the large number of students disengaging with mathematics (Cooper, 2014) and persistent racial, ethnic, and income achievement gaps (NCTM, 2014), research has turned to describing ways to support student engagement. Perhaps no work has been more foundational to our collective understanding of students' cognitive engagement in mathematics than Carol Dweck's work around mindsets (Dweck, 2006; 2014; Dweck & Yeager, 2020). Dweck's work explains that students and teachers can either believe that intellectual abilities are fixed, and there isn't much that can be done to grow intelligence (i.e., fixed mindset), or that intellectual abilities are adaptable and can be cultivated through instruction and experiences (i.e., growth mindset). Studies linking students' and teachers' growth mindsets toward greater student achievement have placed emphasis on strategies for building these mindsets as they pertain to teaching and learning (Boaler, 2015; Boaler et al., 2021; Boaler, Munson, & Williams, 2022; Dweck, 2014).

Research around the construct of grit has also informed understanding related to students' cognitive engagement in mathematics (Duckworth & Quinn, 2009; Duckworth, 2016; Park, Tsukayama, & Duckworth, 2020; White, 2020; White et al., 2022). Grit is defined as a passion and persistence that allows an individual to remain committed over time (Duckworth & Quinn, 2009; Duckworth, 2016). It is the distinguishing factor between those individuals who persevere through trials and adversity and those who do not (White et al., 2022) and it has been connected to the notion of a growth mindset (Duckworth & Quinn, 2009). Within mathematics classrooms, in particular, grit has been captured through the notion of productive struggle. When students engage in productive struggle, they exhibit reasoning and sense-making that allow them to persevere through effortful learning episodes while making sense of problems (NCTM, 2014; Warshauer, 2011). Teachers can support students in productive struggle by anticipating students' struggles before a lesson, giving ample time to work through their struggles, and asking questions to scaffold thinking without eliminating problematic aspects of the task at hand (Warshauer, 2011).

From a content perspective, cognitive engagement might require differentiated support for students based on their current understanding of the concept and related ideas. A multi-tiered systems of support (MTSS) approach uses a framework to offer targeted support based on the needs of individual and small groups of students (Blackburn & Witzel, 2018). Within this approach, students can receive tiered support depending on whether they are on-grade-level (Tier 1), demonstrate slight deficiencies (Tier 2), or indicate significant deficiencies (Tier 3) for the particular content within a lesson or chapter (Blackburn and Witzel, 2018). Targeted support for each tier of students ensures appropriate scaffolds to allow each group to advance their learning, with attention also given to students who may benefit from deepening learning.

When considering students' cognitive engagement with mathematics, it is also important to consider factors related to cognitive load and working memory, as these can influence how students are able to engage cognitively with mathematics (Friso-Van den Bos et al., 2013; Nunes de Santana, Roazzi, & Nobre, 2022; Raghubar, Barnes, & Hecht, 2010). In particular, providing scaffolds and presenting content in a way that reduces students' cognitive load and demands on working memory while preserving opportunities for rich mathematical thinking ensures rich learning opportunities for all students and aligns with calls for access and equity (NCTM, 2014).

Research also provides information related to students' emotional engagement with mathematics. As indicated within the first pillar, a productive disposition is one of five strands of mathematical proficiency (NRC, 2001). In order to build students' proficiency with mathematics, we must not only ensure they are developing sound mathematics but that they are recognizing the utility of mathematics and seeing themselves as "doers" of mathematics. Bishop (2012) defines identity as a "dynamic view of self, negotiated in a specific social context and informed by past history, events, personal narratives, experiences, routines, and ways of participating" (p. 38). In this way, a student's mathematical identity captures how they view themselves relative to mathematics and can be greatly influenced by the classroom environments and teachers with whom they interact. Unfortunately, many students have negative experiences that hinder their mathematical identities and decrease their confidence (Bishop, 2012). Since identities are dynamic and evolving, rich learning experiences that allow students to engage meaningfully in mathematics while interacting with peers through discourse can positively shape even damaged identities (Bishop, 2012; NCTM, 2014; Rubie-Davies, Stephens, & Watson, 2015). Boaler and colleagues found that students may disengage in mathematics classes when they believe their job is to passively memorize rules (Boaler & Greeno, 2000; Boaler, 2006; Boaler & Williams, 2022), likely due to their mathematical identity as a passive information receiver rather than an active, contributing participant.

Social and emotional learning is integral in helping students to develop healthy identities (Mahoney, Durlak, & Weissberg, 2018; Weissberg, Durlak, & Domitrovich, 2015), with attention on five key clusters of social and emotional competencies, namely: self-awareness, self-management, social awareness, relationship skills, and responsible decision making. Developing these competencies supports students in academic success and positive behaviors (Jones & Kahn, 2017), making it important to consider how teaching and learning experiences across all content areas can support students' social and emotional learning.

How is the Pillar Visible within Math & YOU?

To see the foundational role of **Engaging Content** throughout the Math & YOU curriculum, let's look at the ways the program supports students' cognitive and emotional engagement as grounded in research.

Cognitive Engagement

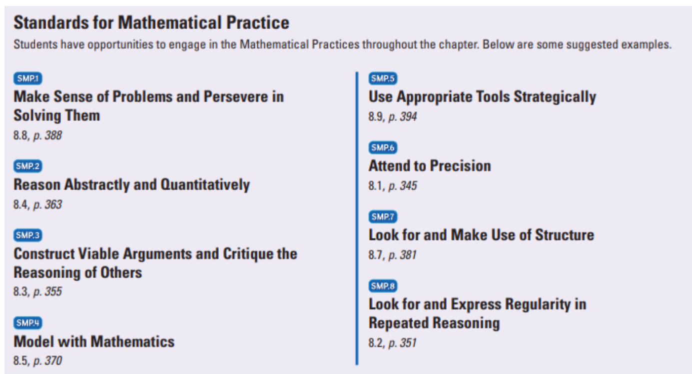
Upon opening the pages of Math & YOU, the student-friendly, visually appealing design of the student lessons is apparent. More than just a hallmark of Dr. Ron Larson's work, the visual design of the student lessons supports student's cognitive engagement by balancing content and whitespace while ensuring concise instructional text, using color and other indicators to avert students' attention, providing example backreferences within exercise sets to offer support, fitting examples to one page to minimize page-flipping, and using photos and images to balance students' cognitive load.

Math & YOU further supports students' cognitive engagement through the student-centered lesson design, in which students take an active role in their learning. As seen in Pillar I, student-friendly Learning Targets and Success Criteria are communicated at the outset of each lesson to help students' focus and monitor their learning. The Investigate activity at the start of each lesson positions students as active participants right out of the gate. Throughout each lesson, reminders and tools are provided for students to self-assess where they are in their understanding of the success criteria, with Talk About It features woven through each lesson to support teachers in class discourse to engage students in their conceptual development.

The standards for mathematical practice, or SMPs, are important processes foundational to mathematical reasoning (NGA & CCSSO, 2010), and provide a great start for ensuring that students are cognitively engaged. Math & YOU was developed to ensure students are engaging in multiple SMPs throughout each lesson. The Standards for Mathematical Practice codes (SMP.1-SMP.8) are explicitly labeled in the Student Edition, to aid students with taking ownership of opportunities and ways to develop the math practice skills as they learn. The Standards for Mathematical Practice feature at the start of each chapter provides one specific moment within each chapter where each SMP is addressed (see Figure 4), supporting teachers in recognizing and capitalizing on these opportunities for cognitive engagement. Indicators throughout the lessons' Author's Notes indicate additional opportunities to engage students in the SMPs and offer guiding questions and support for the teacher to facilitate that engagement. For instance, the note provided in Figure 5 indicates to the teacher an opportunity for students to engage in SMP 6, "Attend to precision." The question prompt provided helps the teacher to support students' engagement in precision with questions that facilitate students' attention on the importance of units and relating this to a strategy for adding fractions.

Figure 4

An Example of How Each Standard for Mathematical Practice is Addressed within the Chapter is Provided to Start the Chapter (Grade 4 TE Vol. 2, p. 341C)



Standards for Mathematical Practice

Students have opportunities to engage in the Mathematical Practices throughout the chapter. Below are some suggested examples.

SMP.1 **Make Sense of Problems and Persevere in Solving Them** 8.8, *p. 388*

SMP.2 **Reason Abstractly and Quantitatively** 8.4, *p. 363*

SMP.3 **Construct Viable Arguments and Critique the Reasoning of Others** 8.3, *p. 355*

SMP.4 **Model with Mathematics** 8.5, *p. 370*

SMP.5 **Use Appropriate Tools Strategically** 8.9, *p. 394*

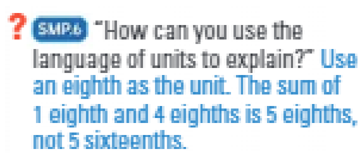
SMP.6 **Attend to Precision** 8.1, *p. 345*

SMP.7 **Look for and Make Use of Structure** 8.7, *p. 381*

SMP.8 **Look for and Express Regularity in Repeated Reasoning** 8.2, *p. 351*

Figure 5

Author's Notes within Lessons Support Teachers in Engaging Students in Standards for Mathematical Practice (Grade 4 TE Vol. 2, p. 345)



? SMP.6 "How can you use the language of units to explain?" Use an eighth as the unit. The sum of 1 eighth and 4 eighths is 5 eighths, not 5 sixteenths.

The English Learner Support feature is interspersed throughout the lessons and provides additional information and resources for the teacher to ensure adequate steps are taken to support English language learners' content engagement. Students may struggle with the content not because they lack an understanding of mathematics, but because they are having a difficult time translating the mathematical language being used during the lesson. Supports like the one illustrated in Figure 6 help teachers to identify instances where language may be a barrier to students' mathematical learning. Furthermore, these notes offer pedagogical strategies for helping students work through these challenges while holding them to high expectations of learning. Many of the English Learner Support notes offer differentiated support based on a learner's level of language proficiency, offering separate supports for students who are Entering-Emerging, Developing-Expanding, or Bridging-Reaching (WIDA, 2020).

Figure 6

English Learner Support Provides Strategies to Help Cognitively Engage English Learners (Grade 4 TE Vol. 2, p. 342)

EL English Learner Support

Constructing Mathematical Explanations

On the board, write the review words and the words *fifths, hundredths, tenths, thirds,* and *twelfths*. Ask EL students how many equal parts of a whole each word represents. For each word, have them complete and read aloud the statement "The word _____ represents _____ equal parts of a whole."

Each lesson wraps up with a Supporting Student Learning feature (see Figure 7). This feature provides information to help teachers engage students at different levels of understanding of the material from the current lesson using the MTSS tiers of support (Blackburn and Witzel, 2018). Providing this support helps the teacher to differentiate instruction so that all students can remain actively engaged and progress toward the common learning target with cognitively appropriate support.

Figure 7

Supporting Student Learning Feature that Provides Next Steps for Students' Appropriate Cognitive Engagement (Grade 6 TE, p. 54A)

Supporting Student Learning

Use the **Quick Check** exercises to assess understanding of key concepts of the lesson, and support emerging learners with additional resources. Remember to check students' **Self-Assessment** to help inform your instructional decisions.

Reinforce (TIER 1)

Identify where students need support with the **key learning of this lesson**.

- *Exercise 1:* Draw a model to explain fraction addition and subtraction.
- *Exercise 2:* Add and subtract fractions.
- *Exercise 3:* Write a mixed number as an improper fraction.
- *Exercise 4:* Add and subtract mixed numbers.

Re-engage (TIER 2)

Identify where students need support with **prior skills**.

- Add and subtract mixed numbers with like denominators. *Grade 4 Sections 8.7 and 8.8*

Use your diagnostic tools to gain insight into what prior skills students know.

Intervene (TIER 3)

Identify where students have additional **unfinished learning**.

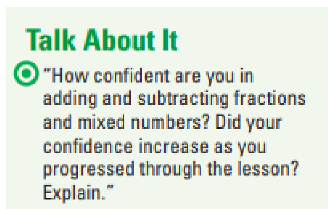
- Use multiplication and division to find equivalent fractions. *Grade 4 Sections 7.3 and 7.4*

Emotional Engagement

Positively engaging students in their mathematical learning experiences was a focus throughout development of the Math & YOU program. To begin, a focal point of the program development included ensuring that all students saw themselves represented in Math & YOU. Imagery throughout the program was chosen to challenge common stereotypes based on gender and disabilities. The text consistently uses "you" within problem contexts rather than proper names of hypothetical people so that students feel engaged and invested in the mathematics they are thinking about. Students can also see themselves as capable of mathematics through their opportunities to self-reflect (recall Figure 3c; see Figure 8) and messaging that promotes growth mindsets through portrayal of mathematics as a learning subject rather than a performance subject (Boaler, 2015). In fact, self-reflection opportunities (recall Figure 3c) are intentionally worded to encourage students' growth mindsets, allowing them to rate themselves from the following options that promote views of mathematics as a learning-subject: *(1) I don't understand yet. (2) I can do it with help. (3) I can do it on my own. (4) I can teach someone.*

Figure 8

Talk About It Connects Back to the Lesson Learning Target and Provides an Opportunity for Self-Reflection (Grade 6 TE, p. 54)



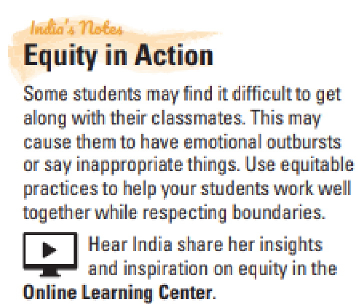
Talk About It

"How confident are you in adding and subtracting fractions and mixed numbers? Did your confidence increase as you progressed through the lesson? Explain."

Across all grade levels, students' emotional engagement is supported through various notes and additional videos offered by Dr. India White (White, 2000; White et al., 2022). Equity in Action notes support teachers in engaging students in ways that extend beyond the mathematical content by bringing to light important elements of access and equity and considerations for supporting students with a diversity of backgrounds. The example Equity in Action note in Figure 9 supports teachers in recognizing behavioral and emotional aspects that might influence students' engagement and provides suggestions for how teachers might support students with equitable teaching practices. Videos provided in the Online Learning Center offer additional information and support for teachers in ensuring they establish equitable practices that will promote learning for students who may face barriers due to race, socioeconomic status, or other factors.

Figure 9

India's Equity in Action Notes Support Teachers in Considerations of Access and Equity that Might Influence Student Engagement (Grade 6 TE, p. 54A)



India's Notes

Equity in Action

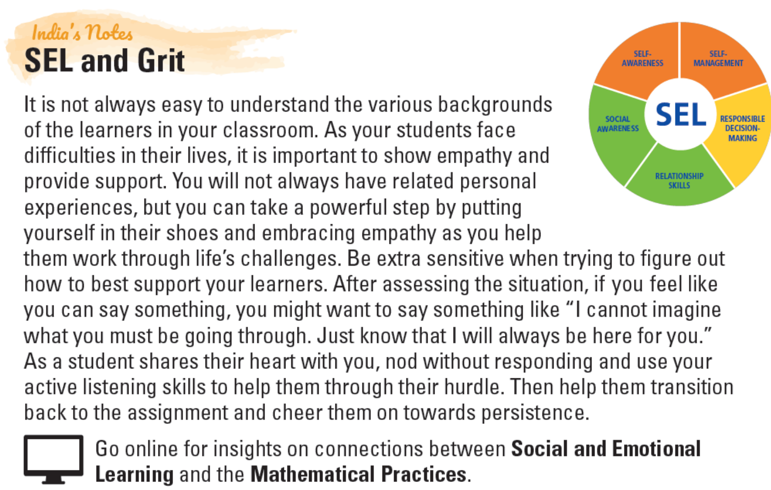
Some students may find it difficult to get along with their classmates. This may cause them to have emotional outbursts or say inappropriate things. Use equitable practices to help your students work well together while respecting boundaries.

Hear India share her insights and inspiration on equity in the **Online Learning Center**.

Further teacher support comes in India's SEL and Grit feature throughout Math & YOU and illustrated in Figure 10. This section supports teachers in helping students to develop the five components of Social and Emotional Learning (Mahoney et al., 2018; Weissberg et al., 2015). Tips also empower teachers in supporting students in developing grit through persistence in problem solving. Additional video features provide information that connects this learning to the SMPs, a powerful way to simultaneously engage students cognitively and emotionally.

Figure 10

India's SEL and Grit Notes Support Students' Development Along the 5 SEL Constructs (Grade 4 TE Vol. 2, p. 341D)



India's Notes

SEL and Grit

It is not always easy to understand the various backgrounds of the learners in your classroom. As your students face difficulties in their lives, it is important to show empathy and provide support. You will not always have related personal experiences, but you can take a powerful step by putting yourself in their shoes and embracing empathy as you help them work through life's challenges. Be extra sensitive when trying to figure out how to best support your learners. After assessing the situation, if you feel like you can say something, you might want to say something like "I cannot imagine what you must be going through. Just know that I will always be here for you." As a student shares their heart with you, nod without responding and use your active listening skills to help them through their hurdle. Then help them transition back to the assignment and cheer them on towards persistence.

Go online for insights on connections between **Social and Emotional Learning** and the **Mathematical Practices**.

Another way Math & You is developed with engaging content is through a consistent theme that ties the mathematics learned to various careers, with a different career highlighted in each Chapter Opener. India's Talk About Careers feature further supports students' engagement in the mathematics of the chapter by introducing a career and providing questions to help pique students' curiosity in the career and how it relates to mathematics, such as the example related to computer programing illustrated in Figure 11. Each chapter showcases an actual professional and includes an interview to showcase how the individual utilizes math in their career. Careful thought was given to making sure gender and racial stereotypes were challenged through the professionals highlighted across the grade, helping students form perceptions of mathematics that incorporate people "like them" as doers of mathematics. With the goal of being aspirational, these career themes open students' experiences to a variety of professions that they may not have considered or been exposed to in their everyday lives.

Figure 11

India's Talk About Careers Notes Support Student Engagement by Making Mathematics Relevant through Connections to Various Careers (Grade 4 TE Vol. 2, p. 341)

India's Notes

Talk About Careers

"Computer programmers are professionals who write computer programs. They test computer programs for errors, fix computer codes that have faults, and create scripts for software."

? "Do you like to work on computers? Do you like to fix things? If so, computer programming might be the perfect career for you!"

Pillar III: Teaching Support

Not only does the Math & YOU program offer strong mathematical content delivered in lessons designed to engage students, but the program also provides **Teaching Support** that sets it apart from other programs. Ultimately, teachers using the Math & YOU program will make decisions during the lesson planning and enactment phases that will transform the intended curriculum to the enacted curriculum (Ball & Forzani, 2011; Stein, Remillard, & Smith, 2007). The Math & YOU program is dedicated to empowering teachers by providing background knowledge, tools and strategies that will equip them to use the program in their own contexts to best meet the needs of the students in their classrooms.

> What students learn "depends fundamentally on what happens inside the classroom as teachers and learners interact over the curriculum."
>
> -Ball and Forzani, 2011, p. 17

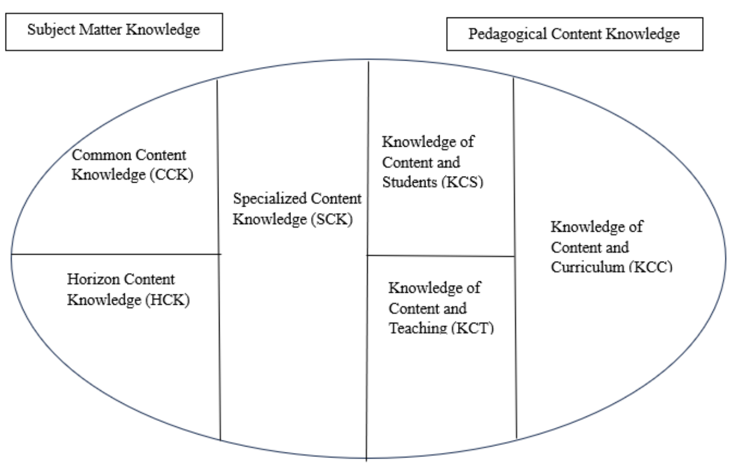
What Research Informs the Pillar?

A breadth of research makes it very clear; teachers play a critical part in the learning that occurs in a classroom (Ball & Forzani, 2009, 2011; Hattie, 2023, 2012; Hattie, Fisher, & Frey, 2017; NCTM, 2014; Stein et al., 2007). Research has described the ways teachers interact with the curriculum to influence student learning outcomes through lesson planning and lesson implementation (Silver & Stein, 1996; Smith & Stein, 1998; Stein et al., 2007). Decisions about selecting and sequencing activities and tasks, as well as in-the-moment decisions based on student interactions during live instruction will ultimately influence the student learning that occurs through implementation of a curriculum (Stein et al., 2007). The resulting learning may or may not be aligned with the intended learning. Teacher knowledge is a key contributing factor in accounting for the decisions made by teachers during lesson planning and enactment that ultimately shape student learning (Stein et al., 2007). Other factors include, but are not limited to, the teachers' beliefs, classroom structures and norms, and broader organizational and policy contexts (Stein et al., 2007).

Given its documented role in teacher decision making, many researchers have tried to provide an account of the collective knowledge required to teach mathematics. Deborah Ball and colleagues (Lowenberg Ball, Thames, & Phelps, 2008) expanded upon Lee Shulman's (1986) prior descriptions of pedagogical knowledge to provide a highly accepted description of the nuanced Mathematical Knowledge for Teaching (MKT). The resulting model, illustrated in Figure 12, distinguishes between subject matter knowledge and pedagogical content knowledge (Lowenberg Ball, Thames, & Phelps, 2008). Furthermore, subject matter knowledge (SMK) is broken into common content knowledge (CCK; i.e., what any adult might be expected to know of a mathematical concept), horizon content knowledge (HCK; i.e., knowledge “beyond” the content at hand to understand where students are headed with their understanding), and specialized content knowledge (SCK; i.e., knowledge about various strategies or multiple representations that facilitate instruction). Pedagogical Content Knowledge (PCK) is also broken into three sub-categories, including knowledge of content and students (KCS; i.e., likely misconceptions and students’ preferred representations), knowledge of content and teaching (KCT; i.e., mathematics- and even concept-specific pedagogical approaches and strategies as well as effective sequencing of ideas and examples), and knowledge of content and curriculum (KCC; i.e., how mathematical ideas fit together across the curriculum to build a comprehensive mathematical picture). The complexity of MKT highlights both the large body of knowledge required to teach mathematics and the many ways curricula materials might support teachers’ planning and enactment of instruction by bringing important aspects of these knowledge domains to light.

Figure 12

Mathematical Knowledge for Teaching Domains Described by Deborah Ball (Ball, Thames, & Phelps, 2008)



One particularly useful way to think about the in-the-moment decision making of teachers is through the lens of professional noticing (Lamb & Phillip, 2010; van Es & Sherin, 2021). The professional noticing framework describes the complexity of teachers *attending* to student thinking while engaging on a mathematical task, *interpreting* the student thinking to draw conclusions about what the student knows and understands and what remains to be learned, and then *responding* (Lamb & Phillip, 2010) or *shaping* (van Es & Sherin, 2021) the next appropriate instructional step to move student thinking forward and/or gain additional interactions for *interpretation*.

There are also several research-based descriptions of pedagogical strategies that are most effective in classrooms generally (Hattie, 2023, 2012, Hattie & Timperley, 2007; Specjal & Hattie, 2023) and mathematics classrooms, specifically (Hattie, Fisher, & Frey, 2017; NCTM, 2014). John Hattie's ten high impact teaching strategies emerged from synthesis of thousands of studies that measured the effect size of interventions on student learning. The ten high impact teaching strategies emerged as having greater than average effect size, and include: setting goals, structuring lessons, explicit teaching, worked examples, collaborative learning, multiple exposures, questioning, feedback, metacognitive strategies, and differentiated teaching (Hattie, 2023, 2012; Hattie, Fisher, & Frey, 2017; Hattie & Timperley, 2007). NCTM (2014) published eight effective teaching practices supported by research on effective learning in mathematics specific classrooms, namely: establish math goals to focus learning, implement tasks that promote reasoning and problem solving, use and connect mathematical representations, facilitate meaningful mathematical discourse, pose purposeful questions, build procedural fluency from conceptual understanding, support productive struggle in learning mathematics, and elicit and use evidence of student thinking. Together, these two frameworks describe teaching practices which support students' learning and can inform lesson structure and teaching supports within curricular materials to encourage instruction most likely to lead to student growth and learning.

Teacher reflection is widely supported as an important element of a teacher's professional growth. The Danielson Framework for Teaching (Danielson Group, 2023) highlights growing and developing professionally as a component of "Principled Teaching." A key aspect of growing and developing professionally is seeking and acting on feedback, including personal reflections on lessons. A model of professional development experiences that support change in teaching practice illustrates the importance of opportunities for teachers to reflect on their in-class experiences and the outcomes of those experiences (Hill & Papay, 2022). Thus, it is important to embed such experiences for teachers within curriculum.

How is the Pillar Visible within Math & YOU?

The Math & You program provides targeted support to empower teachers with the subject matter knowledge and pedagogical content knowledge to deliver effective lessons.

Subject Matter Knowledge

Several features in the Math & YOU program provide subject matter support for teachers that span the three domains of subject matter knowledge described by Ball and colleagues (2008). Recall that each chapter provides a <u>Coherence Through the Chapter</u> and a <u>Coherence through the Grades</u> feature (see Fig. 3). <u>Coherence Through the Chapter</u> outlines the content standards for the current chapter (CCK) while <u>Coherence Through the Grades</u> outlines the prior and future content (HCK) that builds on the ideas learned in the chapter.

Each chapter, regardless of grade level, begins with a detailed <u>Author's Notes Overview</u> organized around three sections: *What We're Doing*, *Why We're Doing It*, and *Essential Background*. As seen from the sample in Figure 13, these <u>Overviews</u> provide additional subject matter knowledge information for teachers. *What We're Doing* provides a big picture of the key concepts to be learned in the section (CCK), providing details related to the role of specific representations or strategies (SCK) that might be important to highlight. *Why We're Doing It* outlines any applications or real-world connections related to the content (SCK) as well as describing future concepts which require a strong foundation in the current learning (HCK).

Figure 13

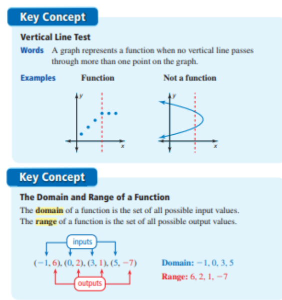
Author's Notes Overview Provides Important Content Knowledge and Pedagogical Content Knowledge for Teachers (Algebra 1 TE Instructional Guide, p. 88)

Paul's Notes

Overview

What we're doing…

Chapter 3 begins by investigating the concept of a function with emphasis on identification of the domain and range of a function. Students will see functions in multiple representations: analytically, graphically, numerically, and verbally. Throughout the chapter, students will look deeper at the characteristics of a function, identify intercepts and maximum and minimum values, identify the constant rate of change of linear functions, and learn to distinguish linear from nonlinear functions by analyzing tables and graphs. Toward the end of the chapter, students examine graphing linear functions in standard form and slope-intercept form and build knowledge of absolute value equations by showing students how to graph absolute value functions.



Why we're doing it…

Many real-life quantities can be classified as linear functions. For this reason, an understanding and appreciation for the slope of a function is a central theme of the chapter. Additionally, other concepts in this chapter lay the foundation for a greater understanding of a variety of functions and their graphs. The rules being applied to a parent function can be applied across all functions, so this skill will be an important one to master.

Essential Background

Prior to this chapter, students should be proficient in solving literal equations and evaluating expressions. It is important for students to be familiar with the coordinate plane, plotting points, and basic graphing techniques because graphing will be a focus in this chapter.

Each lesson begins with a Focus feature to make explicit the content to be learned within the lesson (CCK) through communication of a lesson Learning Target and Succes Criteria. The Lesson Coherence feature situates the content from the lesson within students' previous and future learning, but also highlights representations or strategies developed through the current lesson (SCK). Examples of Lesson Focus and Coherence are provided in Figure 14.

Figure 14

Focus and Coherence Features at the Lesson Level Support Teachers' Subject Matter Knowledge (Grade 6 TE p. 51A)

Focus

Learning Target:
Add and subtract fractions and mixed numbers.

Success Criteria:
- Draw a model to explain fraction addition and subtraction.
- Add and subtract fractions.
- Write a mixed number as an improper fraction.
- Add and subtract mixed numbers.

Coherence

In previous grades, students learned to add and subtract fractions and mixed numbers with like and unlike denominators. This section reviews and refreshes these concepts so that students can apply this understanding to adding and subtracting decimals.

This section also provides an opportunity to review drawing models to represent fractions and mixed numbers. In future sections, these models will support students' understanding of multiplying and dividing fractions and mixed numbers.

Prerequisite Skills

Grade 4 Sections 8.7 and 8.8: Add and subtract mixed numbers with like denominators.

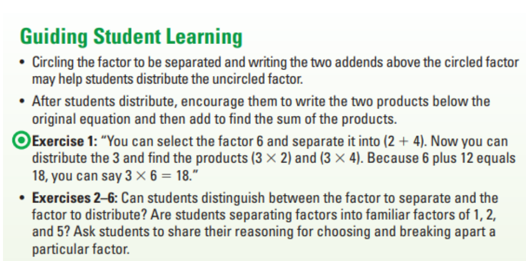
Pedagogical Content Knowledge

The Math & YOU program provides considerable attention to developing teachers' pedagogical content knowledge through a variety of features. Teachers' knowledge of content and curriculum (KCC) is supported by the Coherence Through the Chapter, Coherence Through the Grades, and Lesson Coherence features that were highlighted in Figures 3 and 14. The *Why We're Doing It* and *Essential Background* sections of the Author's Notes (see Figure 13) also provide context to the teacher about how the current learning fits into the larger picture, emphasizing how the current learning will be important for future learning as well as the past learning that is foundational to developing a robust understanding in the current chapter. Each of these features related to coherence describes how the current learning fits into the broader curriculum at various scales, from within the lesson itself, to across multiple lessons in the chapter, to across multiple grade-levels. These supports help the teacher to always situate the current learning within the context of students' developing mathematical understanding on a larger scale (KCC). This helps students build a connected, complete picture of mathematics rather than developing incomplete and disjointed ideas.

Knowledge of content and students (KCS) supports are included throughout the Math & YOU program. Guiding Student Learning notes and Author's Notes (see Fig. 15) in the teacher's edition highlight key aspects of KCS, indicating representations and intermediate steps that might scaffold student thinking and support their organization. The Guiding Student Learning note in Figure 15a provides notational supports that might help students making a common mistake of forgetting to distribute a factor. Laurie's Connect to Data note in Figure 15b brings to light a strategy that students are likely to use (i.e., *doubles plus 1 strategy*) and encourages teachers to challenge students to incorporate a different strategy (i.e., *doubles minus 1 strategy*).

Figure 15

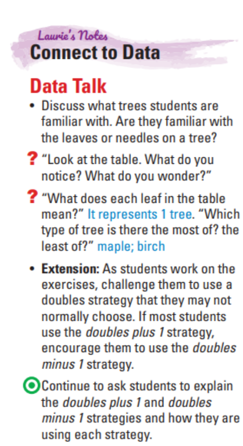
Various Features Support Teachers' Knowledge of Content and Students by Highlighting Common Strategies and Supports to Help Students Organize Their Thinking (Grade 3 TE Vol. 1 p. 130 and Grade 2 TE Vol. 1 p. 50)



Guiding Student Learning

- Circling the factor to be separated and writing the two addends above the circled factor may help students distribute the uncircled factor.
- After students distribute, encourage them to write the two products below the original equation and then add to find the sum of the products.
- **Exercise 1:** "You can select the factor 6 and separate it into $(2 + 4)$. Now you can distribute the 3 and find the products (3×2) and (3×4). Because 6 plus 12 equals 18, you can say $3 \times 6 = 18$."
- **Exercises 2–6:** Can students distinguish between the factor to separate and the factor to distribute? Are students separating factors into familiar factors of 1, 2, and 5? Ask students to share their reasoning for choosing and breaking apart a particular factor.

Fig 15a. Guiding Student Learning



Laurie's Notes

Connect to Data

Data Talk

- Discuss what trees students are familiar with. Are they familiar with the leaves or needles on a tree?
- ? "Look at the table. What do you notice? What do you wonder?"
- ? "What does each leaf in the table mean?" It represents 1 tree. "Which type of tree is there the most of? the least of?" maple; birch
- **Extension:** As students work on the exercises, challenge them to use a doubles strategy that they may not normally choose. If most students use the *doubles plus 1* strategy, encourage them to use the *doubles minus 1* strategy.
- Continue to ask students to explain the *doubles plus 1* and *doubles minus 1* strategies and how they are using each strategy.

Fig 15b. Authors' Notes

Authors' Notes (see Fig. 16) also frequently indicate common student errors or misconceptions, as seen in Paul's Other Characteristics of Functions note which highlights students' common focus on *y*-coordinates rather than *x*-coordinates when solving the problem in Fig. 16a. Author's Notes also support knowledge of content and teaching (KCT) by indicating models and examples that might be particularly relevant at particular moments in instruction, as illustrated in Nick's Lesson Insights illustrated in Fig. 16b.

Figure 16

Author's Notes (Algebra 1 TE Instructional Guide p. 88 and Grade 8 TE p. 16) Support Knowledge of Content and Students by Highlighting Frequent Student Errors and Offering Pedagogical Supports

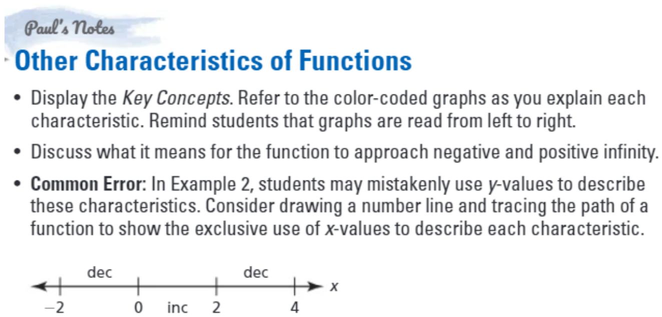


Fig. 16a. *Author's Note Supports KCS*

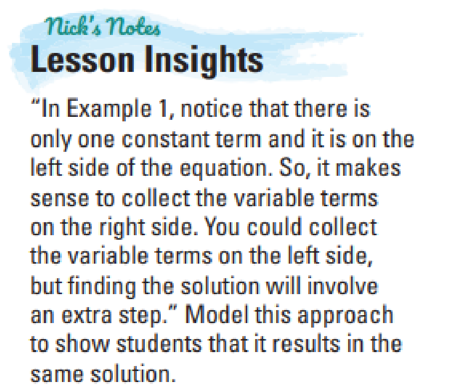


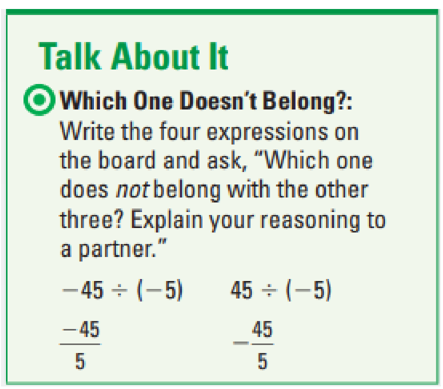
Fig 16b. *Author's Note Supports KCT*

Math & YOU supports teachers' knowledge of content and teaching both through reinforcing research-based, effective pedagogical strategies through the lesson design and structure, as well as providing additional supports throughout the teaching editions to support teachers in pedagogical approaches at specific instances of instruction by providing "just in time" notes at the point of use. While developing lesson structure and designing teacher supports, the Math & YOU authors focused on building teaching environments in which teachers could leverage Hattie's high-impact teaching strategies (Hattie, 2012; 2023) and NCTM's eight effective teaching strategies (NCTM, 2014).

For brevity, we illustrate how just a few of these teaching practices are supported. Math & You has an obvious focus on "teacher clarity" (d=0.75; Hattie, 2012, 2023) or "establish mathematics goals to focus learning" (NCTM, 2014) through a conscious effort to frame all learning with clear and visible Learning Targets and Success Criteria. These have been developed to move students from surface to deep level understandings and to support students' ability to transfer understandings from one concept to another (Biggs & Collis, 1991, Specjal & Hattie, 2023). This emphasis on the learning target and success criteria is visible in both the student and teacher editions of the materials. Students have opportunities to self-reflect on their progress toward meeting the lesson success criteria and specific problems are tied to criteria to support students in that self-reflection. But the teacher edition also shows that the learning target and success criteria are not just stated at the beginning of the lesson but form a cohesive guide throughout the lesson to support the teacher in making clear what learning is expected from the lesson and helping students to monitor progress along the way. "Classroom discussion" (n=0.82; Hattie, 2012, 2023), which relates to "facilitate meaningful mathematical discourse" (NCTM, 2014), and "feedback" (0.73; Hattie, 2017; Hattie & Timperley, 2007), which relates to "elicit and use evidence of student thinking" also permeate the curriculum, with Talk About It activities and Math Talks that include prompts likely to encourage rich discussions between students and frequent and insightful opportunities for feedback to help guide learning and teaching (Smith & Stein, 2018; Specjal, 2022). The Sample Talk About It in Figure 17 showcases an opportunity to engage 7th grade students in the *Which One Doesn't Belong* instructional routine to encourage discussion between students while providing evidence of student thinking on which the instructor can base additional discussions (Dweck, 2014).

Figure 17

Sample Talk About It Encourages Research-Based Teaching Practices by Promoting Discussion and Eliciting Student Thinking through the Which One Doesn't Belong Strategy (*Grade 7 TE, p. 54)*


Talk About It

Which One Doesn't Belong?: Write the four expressions on the board and ask, "Which one does *not* belong with the other three? Explain your reasoning to a partner."

$-45 \div (-5)$ $45 \div (-5)$

$\frac{-45}{5}$ $-\frac{45}{5}$

In addition to the natural alignment between the instructional approaches and various research-based teaching practices, the Math & You series provides support, training, and resources for teachers throughout. Highlights include author digital videos of the "what" and "why" of the chapters, a Support for ALL Learners feature to provide resources for offering tiered support, and point-of-use callouts in the teaching edition to provide readily available support for teachers as they implement a lesson. The many point-of-use notes provided in the Teacher Editions support teachers in the act of professional noticing by pointing out key aspects of student thinking to attend to and possible responses when they are present. These supports empower teachers in the decision-making process to deliver lessons that uphold the desired learning outcomes (Stein et al., 2007).

Teachers are encouraged to reflect at both the lesson level, with a My Thoughts on the Lesson feature using the online platform, and at the chapter level, with a My Thoughts on the Chapter feature. These features encourage continued growth and development across all aspects of teachers' practices (Danielson Group, 2023; NCTM, 2014).

Pillar IV: Innovative Platform

Taken together, the pillars of **Conceptual Foundation**, **Engaging Content**, and **Teacher Support** establish a strong focus and illustrate the Math & YOU commitment to empowering teachers and engaging learners to build a solid mathematical understanding for all students. The final pillar, **Innovative Platform**, strengthens each of the previous pillars through digital enhancements and connections that allow for synergy across the program.

> "An excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ides, reason mathematically, and communicate their mathematical thinking."
>
> -NCTM, 2014, p. 78

What Research Informs the Pillar?

NCTM identifies Tools and Technology as one of six overarching Principles of School Mathematics, together with Teaching & Learning, Assessment, Curriculum, Access & Equity, and Professionalism (NCTM, 2014). By identifying technology as a principle, NCTM is declaring technology should be regularly and purposefully integrated, "indispensable features" (NCTM, 2014) of the classroom.

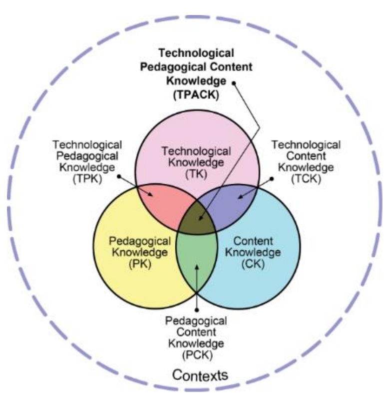
For many years, research around technology in mathematics focused on specific technologies, which could be classified as either conveyance or mathematical action technologies. Dick and Hollebrand (2011) described conveyance technologies as tools that could be used to convey or transmit information within a lesson. Information might be conveyed for a variety of purposes, such as presentation, collaboration, and assessment, but "conveyance technologies are not mathematics specific" (Dick & Hollebrand, 2011, p. 12). Examples of such technologies might include presentation software, discussion boards, and polling apps. On the other hand, mathematical action technologies "can perform mathematical tasks and/or respond to the user's actions in mathematically defined ways" (Dick & Hollebrand, 2011, p. 12). Examples of such technologies include spreadsheets, graphing calculators, and dynamic geometry software. Both conveyance and mathematical action technologies are important for the mathematics classroom.

One particularly interesting, and growing use of technology in the classroom, is providing data to support data-driven decision making (Marsh, Pane, & Hamilton, 2006). For teachers, implementing data-driven instruction requires interpreting data to identify strengths and weaknesses in progress toward meeting specific learning objectives and then using that knowledge to design future instruction (Dunn et al., 2013; Pella, 2012). But grounding classroom practices and instructional decisions around student thinking is a complex task (Franke, Kazemi, & Battey, 2007). Technology can be leveraged as a tool to support teachers in the effective teaching practice of "eliciting and using evidence of student thinking" (NCTM, 2014) and the high leverage practice "provide feedback to assist students in advancing their learning" (Hattie, 2009; Marzano, 2007; NCTM, 2014) because of its capability to provide frequent and immediate feedback. This is well aligned with NCTM's (2014) Assessment Principle, which asserts the importance of making deliberate use of data as evidence of learning to make instructional decisions. In order to provide the information needed to make meaningful interpretations of student thinking, assessments must be actionable (Hess, 2023). Hess defines an actionable assessment as one that uncovers student thinking and tells us where students are on their learning trajectory (Hess, 2023). Furthermore, Hess (2023) describes how this can be used to inform next steps through an actionable assessment cycle that uses new information to inform next instructional steps in a cyclical process.

Like other aspects of teaching, teaching with technology is complex and requires teachers to apply a breadth of knowledge. Mishra and Koehler (2006) first developed the Technological, Pedagogical, and Content Knowledge (TPACK) Framework by expanding upon Shulmans' (1986) work around teacher knowledge cited in the **Supporting Teachers** pillar. The TPACK model highlights that when using technology, technical knowledge interacts with pedagogical knowledge (Technological Pedagogical Knowledge, TPK) and with content knowledge (Technological Content Knowledge, TCK). Furthermore, all three intersect to define Technological Pedagogical Content Knowledge (TPACK), as illustrated in Figure 18. This model highlights the complexity of teaching with technology but also illustrates the interplay between technology and other aspects of teaching and learning. Curricula can support teachers by providing integrated experiences that leverage technology while supporting the use of effective pedagogical practices that build students' understanding of conceptual mathematics.

Figure 18

TPACK Framework Evolving from the work of Mishra and Koehler (2006) Shows Interactions Between Technological Knowledge, Pedagogical Knowledge, and Content Knowledge



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Using data to inform instructional decisions can be particularly complicated and requires teachers to translate information provided by the system into meaningful knowledge which can then guide pedagogical decision-making (Kitto, Buckingham Shum, & Gibson, 2018; Wise & Vytasek, 2017). Research around the use of learning analytics has investigated how student data can support an iterative assessment cycle that informs future learning activities (Mor, Ferguson, & Wasson, 2015) such as the one described by Hess (2022). Researchers have described two different categories of information teachers might use during the lesson design process, namely checkpoint analytics (i.e., do students access the materials?) and process analytics (i.e., how do students complete the tasks?) (Lockyer, Heathcote, & Dawson, 2013). Research has also focused on how teachers use analytics in real-time to adapt instruction according to students' needs (Dillenbourg, 2013), including how it can support scaffolding for whole-class and small groups of students (Tan, Koh, & Jonathan, 2018). Wise and Jung (2019) similarly found that actions taken by teachers in response to analytics aligned with three categories: whole-class scaffolding, targeted scaffolding, and revising course elements. Their work found that it is important to assist teachers in linking their pedagogical questions with the data-informed answers and corresponding appropriate pedagogical responses to leverage the usefulness of data analytics platforms (Wise & Jung, 2019).

How is the Pillar Visible within Math & YOU?

Math & YOU features a brand-new, innovative digital experience that supports a strong **Conceptual Foundation** while providing an **Engaging** learning environment for students and **Supporting Teachers** with the tools and resources they need throughout the planning, teaching, and assessment cycle. In this way, it brings all the pillars together by strengthening and connecting these elements while building on research related to pedagogy with technology and use of data to inform instruction. Let's look at how the digital platform strengthens the effectiveness of each of the other pillars.

Conceptual Foundation and Supporting Teachers

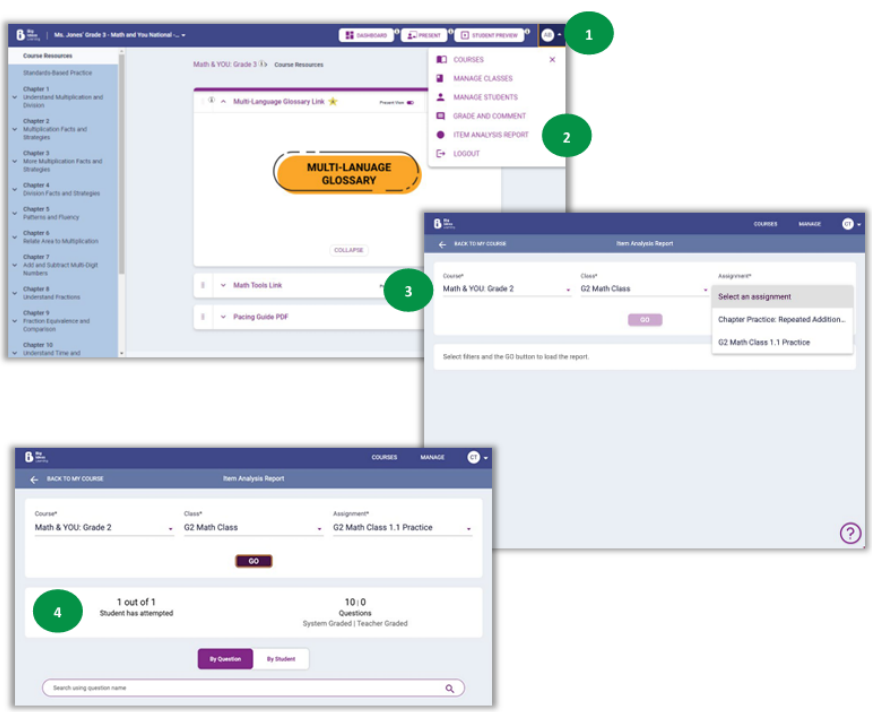
It is hard to separate these two pillars when considering the impact of the digital platform, since the functionality and features of the digital platform **Support Teachers** in building a stronger **Conceptual Foundation** for each student by providing data-informed suggestions for additional learning experiences and differentiated scaffolds to support students' learning. For that reason, we consider how the **Innovative Platform** supports and strengthens these two pillars simultaneously.

The digital platform brings all the resources a teacher needs for lesson planning in one central location, with the ability to customize as needed. Using the Plan mode, a teacher can review all the lesson materials available in the physical editions with the ability to select and sequence activities in the way that best fits the needs of the teacher and her students. After building a lesson to fit students' particular needs, the teacher can use Present mode to share the lesson in full-screen teaching mode for whole class, small-group, or individual instruction as appropriate. Digitally interactive versions of the same content included within the student textbook are available in the digital platform. This includes assessments, which can also be customized through reordering and addition and deletion of questions, supporting engagement of students at the appropriate cognitive level.

But the digital tool is much more than a planning and presenting tool. The digital experience will provide a great deal of data, including both checkpoint analytics and process analytics (Lockyer et al., 2013). Perhaps most meaningful for informing daily instruction, the system provides analytics to **Support Teachers** in interpreting results of assessment. Driven by pre-assessments, formative assessments throughout the learning process, and summative assessments at distinct learning posts, the digital platform provides data for decision making both in-the-moment and for planning future instruction (Dillenbourg, 2013; Stein et al., 2007). Using the Dashboard, the teacher can access reports on how individual students and groups of students have performed on past assessments. Figure 19 illustrates how a teacher can navigate to the Item Analysis Report for a particular course, class, and assignment to get a summary of information including how many students have completed the assignment, how many questions are in the assignment, and which questions will be manually versus automatically graded.

Figure 19

Analysis Report Providing Assignment Overview



From here, a teacher can choose to access additional reporting organized either By Question or By Student. These reports provide detailed information, including information about the average time spent, average score, and the percentage of students who answered correctly or incorrectly. The reports support teachers' pedagogical responses to the data by providing aggregate and student-specific data regarding pre-requisite skills and common misconceptions which then feed "next steps" in helping students progress toward the learning targets (Wise & Jung, 2019). Upon student completion of any activity, the Quick Report feature provides formative data and suggested next steps, such as Extra Practice (Proficient Learners), Enrichment and Extension (Advanced Learners), and Skills Trainer (Emergent Learners) experiences. Relative to the student experience, digital versions of the activities such as the Reteach provide answer-specific feedback to provide timely and informative feedback for students (Hattie, 2012). Ultimately, this will lead to stronger **Conceptual Foundations** of the mathematics being studied.

Engaging Content

The Math & YOU digital platform brings the engaging content of the student edition online in a new, interactive format. The digital platform makes use of conveyance technologies to provide an engaging experience using guided interactive lessons that can be customized by the teacher to provide an experience just right for these specific students. The lessons use videos and interactive examples while offering all the same lesson features that were found in the student edition. Investigation activities in the digital platform provide a hands-on approach including models and manipulatives to be used while exploring and discovering. Feedback is personalized and immediate, providing individualized support to engage each student right where they are at. Career Explorations Videos support students' emotional engagement by presenting real professionals who use mathematics in their careers. The mobile-friendly design makes it easy for students to access content in a time and place available to them.

Summary

Informed by research and focused on the four pillars of **Conceptual Foundation**, **Engaging Content**, **Supporting Teachers**, and **Innovative Digital Platform**, Math & YOU is a comprehensive program that recognizes and addresses the complexity of teaching and learning experiences that maximize student learning outcomes. With lessons designed to develop rigorous mathematical understanding from a strong conceptual foundation, content developed to engage all students cognitively and emotionally throughout the lessons, supports that build teachers' pedagogical and subject matter knowledge while emphasizing research-based pedagogical strategies, and a digital experience that provides enhancements to each of these features and supports data-driven instruction, Math & YOU takes a research-grounded approach to supporting YOU along your K-12 mathematical teaching and learning journey.

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